GAW – WCCAP recommendation mobility particle size spectrometers - Part IV Constants and Relevant Equations

Alfred Wiedensohler and Wolfram Birmili

Leibniz Institute for Tropospheric Research

Constants and equations

This recommendation is based on the article of Wiedensohler et al. (2012).

The constants and equations follow the recommendations in the ISO15900 standardization (also given in Kim et al., 2005):

Dynamic gas viscosity at 296.15 K and 1013.25 hPa:

$$\eta_0 = 1.83245 \cdot 10^{-5} \frac{\mathrm{kg}}{\mathrm{m\ s}}$$

$$\eta = \eta_0 \left(\frac{T}{T_0} \right)^{3/2} \left(\frac{T_0 + 110.4\mathrm{K}}{T + 110.4\mathrm{K}} \right)$$

Mean free path at 296.15 K and 1013.25 hPa:

$$\lambda_0 = 67.3\ 10^{-9}\mathrm{m}$$

$$\lambda = \lambda_0 \left(\frac{T}{T_0} \right)^2 \left(\frac{p_0}{p} \right) \left(\frac{T_0 + 110.4\mathrm{K}}{T + 110.4\mathrm{K}} \right)$$

Cunningham correction:

$$C_\mathrm{C} = 1 + \frac{2 \cdot \lambda}{d_\mathrm{P}} \left(1.165 + 0.483 \cdot \exp\left(-0.997 \frac{d_\mathrm{P}}{2 \cdot \lambda} \right) \right)$$

Bipolar charge distribution

To calculate the bipolar charge distribution analytically, an approximation formula for lower charging states, n, (-2,-1, +1, +2) was developed (Wiedensohler, 1988). This formula is valid for particle size ranges from 1 to 1000 nm or 20 to 1000 nm particle diameter for n equal to -1, 0, +1 or -2, +2, respectively. The according approximation coefficients are given in Table 1.

Approximation formula:

$$F(n) = 10^{\left(\sum_{i=0}^{5} a_i(n)\left(\log\frac{D_P}{nm}\right)^i\right)}$$

	Approximation coefficients $a_i(n)$				
i	n=-2	n=-1	n=0	n=+1	n=+2
0	-26.3328	-2.3197	-0.0003	-2.3484	-44.4756
1	35.9044	0.6175	-0.1014	0.6044	79.3772
2	-21.4608	0.6201	0.3073	0.4800	-62.8900
3	7.0867	-0.1105	-0.3372	0.0013	26.4492
4	-1.3088	-0.1260	0.1023	-0.1553	-5.7480
5	0.1051	0.0297	-0.0105	0.0320	0.5049

Table 1: Approximation coefficients after Wiedensohler 1988

For higher n (+3, -3, +4, -4 etc.), the Gunn formula below can be used. A ratio of the electrical mobility of positive to negative ions Z_{I+}/Z_{I-} of 1.4/1.6 was suggested in Wiedensohler 1988.

Gunn (1956) equation:

$$F(n) = \frac{e}{\sqrt{4\pi^2 \cdot \varepsilon_0 \cdot D_P \cdot k \cdot T}} \cdot \exp\left(-\frac{\left(n - \left(\frac{2\pi \cdot \varepsilon_0 \cdot D_P \cdot k \cdot T}{e^2} \right) \ln \frac{Z_{I+}}{Z_{I-}} \right)^2}{\left(\frac{4\pi \cdot \varepsilon_0 \cdot D_P \cdot k \cdot T}{e^2} \right)} \right)$$

References

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